XII - MATHEMATICS, PUBLIC EXAM-2020

PART - I

1) 2) $\pi/6$
2) 2) $\sqrt{7}/2\sqrt{2}$.
3) 3) $k = 1/3$.
4) 3) $2xu$.
5) 4) $(0, \frac{1}{8})$
6) 3) consistent.
7) 2) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$
8) 4) 40
9) 1) exactly n root.
10) 4) undefined.
11) 4) N
12) 4) $\sqrt{10}$
13) 3) 3.
14) 1) 2.
15) 3) xoy plane.
16) 2) $1+i$
17) 3) $3\pi/8$.
18) 3) $\pi/3$.
19) 2) 1, 2.
20) 1) $\tan^{-1}(\frac{1}{2})$.

Part - II

21)
$$\frac{1+i}{1-i} = \frac{(1+i)^2}{1+1} = i$$
and $\frac{1-i}{1+i} = -i$
$$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = i^3 - (-i^3) = -2i \text{ //}$$

H.O.D. Maths Department,
S.R.V. Boys Hr.Sec.School,
Muthukalipatty - 637 401.
Rasipuram(Tk), Namakkal(Dt).

22) $|(1+i)(1+2i)\cdots(1+ni)| = |x+iy|$
$|1+i|\,|1+2i|\cdots|1+ni| = |x+iy|$
$\sqrt{1+1}\ \sqrt{1+4}\cdots\sqrt{1+n^2} = \sqrt{x^2+y^2}$
Squaring on both sides
$2 \cdot 5 \cdot 10 \cdots (1+n^2) = x^2+y^2$

23) $\sin^{-1}\left[\sin\left(\frac{5\pi}{4}\right)\right] = \sin^{-1}\left[\sin(\pi + \pi/4)\right]$
$= \sin^{-1}\left[\sin(-\pi/4)\right]$
$= -\pi/4 \in [-\pi/2, \pi/2]$

24) $\vec{r} = -2\hat{i} + \hat{k}$
$\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$
$\vec{L} = \vec{r} \times \vec{F} = -\hat{i} - 2\hat{k}$
magnitude $= \sqrt{5}$
D.C's $\left(\frac{-1}{\sqrt{5}}, 0, \frac{-2}{\sqrt{5}}\right)$

25) $f(x)$ is continuous in $[\frac{1}{2}, 2]$
$f(x)$ is differentiable in $(\frac{1}{2}, 2)$
$f(\frac{1}{2}) = f(2) = \frac{5}{2}$
By R.T, $f'(c) = 0 \Rightarrow c = \pm 1$
$c = 1 \in (\frac{1}{2}, 2)$ //

26) $df = (2x+3)dx$
$df = (4+3)(0.1)$
$= 7(0.1)$
$df = 0.7$ //

27) $I = \int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)}\,dx \rightarrow (1)$

$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$I = \int_0^{\pi/2} \frac{f(\cos x)}{f(\cos x)+f(\sin x)}\,dx \rightarrow (2)$

(1)+(2) $\Rightarrow 2I = \int_0^{\pi/2} dx \Rightarrow 2I = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2}$

$I = \pi/4$

H.O.D. Maths Dept.
S.R.V. Boys Hr.Sec.School,
Muthukalipatty - 637 401.
Rasipuram(Tk), Namakkal(Dt).

28) $y^2 = 4ax \rightarrow (1)$

diff w.r.t. x

$2yy' = 4a$

$y^2 = 2yy'x$

$y = 2y'x$

29) let e_1 and e_2 be the identity elements

Treating e_1 is the identity

$e_1 * e_2 = e_2 * e_1 = e_2 \rightarrow (1)$

Treating e_2 is the identity

$e_1 * e_2 = e_2 * e_1 = e_1 \rightarrow (2)$

(1) & (2) $e_1 = e_2$

30) $y^2 = -4a(x-h)$

$(h,k) = (2,1)$

$(y-1)^2 = -4a(x-2)$

It passes through $(1,3)$

$4 = -4a(-1)$

$a = 1$

$\therefore (y-1)^2 = -4(x-2)$

Part - C

31) $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$

$A^T = \begin{bmatrix} 2 & 1 \\ 9 & 7 \end{bmatrix} \quad |A^T| = 5$

$|A| = 5$

$A^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -9 \\ -1 & 2 \end{bmatrix}$

$(A^T)^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$

$(A^{-1})^T = (A^T)^{-1}$

32) $\Delta = b^2 - 4ac = 16(p+1)(p-2)$

$\Delta < 0$, if $-1 < p < 2$ (Imaginary)

$\Delta = 0$; if $p = -1$ or $p = 2$ (real)

$\Delta > 0$; if $-\infty < p < -1$ or $2 < p < \infty$ (distinct real roots)

33) $x^2 = -4ay$ (Diagram)

$(-20, -15)$ and $(20, -15)$ lies on the parabola

$4a = \frac{400}{15}$

$3x^2 = -80y$

34) Vector Eqn:

$\vec{r} = (-4\hat{i} + 2\hat{j} - 3\hat{k}) + t(8\hat{i} + 4\hat{j} - 3\hat{k})$

Cartesian Equation:

$\frac{x+4}{8} = \frac{y-2}{4} = \frac{z+3}{-3}$

35) $f'(x) = x^{4/5}\,2(x-4) + (x-4)^2 \frac{4}{5} x^{-1/5}$

$= \frac{(x-4)(14x+6)}{5x^{1/5}}$

$f'(x) = 0 \Rightarrow x = 4, -3/7$

$f'(x)$ does not exists at $x = 0$

Critical numbers are, $0, 4, -3/7$

36) $u = \log(x^3+y^3+z^3)$

$\frac{\partial u}{\partial x} = \frac{3x^2}{x^3+y^3+z^3}$, $\frac{\partial u}{\partial y} = \frac{3y^2}{x^3+y^3+z^3}$

$\frac{\partial u}{\partial z} = \frac{3z^2}{x^3+y^3+z^3}$

$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3(x^2+y^2+z^2)}{x^3+y^3+z^3}$

37) $\sum f(x) = 1$

$K = \frac{1}{30}$

$P(2 < x < 6) = \frac{6}{30} + \frac{5}{30} + \frac{6}{30} = \frac{17}{30}$

38) $K\int_0^1 x(1-x)^{10}\,dx = 1$ $\quad \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$K\int_0^1 (1-x)\,x^{10}\,dx = 1$

$K\int_0^1 (x^{10} - x^{11})\,dx = 1$

$K\left[\frac{x^{11}}{11} - \frac{x^{12}}{12}\right] = 1$

$K\left(\frac{1}{132}\right) = 1$

$K = 132$

H.O.D. Maths Department
S.R.V. Boys Hr.Sec.School
Muthukalipatty - 637 407.
Pasipuram(Tk), Namakkal(Dt).

39)

P	q	¬P	P→q	¬P∨q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore P \to q \equiv \neg P \vee q$

40) A $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\vec{u} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$

$\vec{v} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$

Vector Equation:

Either $\vec{a}$ or $\vec{b}$ and Two parallel vectors $\vec{u}$ & $\vec{v}$

$\vec{r} = \vec{a} + s\vec{u} + t\vec{v}$;

Cartesian Equation: 1 pt + 2 parallel vector

Type: 1

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

(or)

$$\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

Type: 2

2 pt + 1 parallel vector

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \end{vmatrix} = 0$$

(or)

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

41 a) Part-D

$$[A|B] = \begin{bmatrix} 1 & -1 & 1 & -9 \\ 2 & -1 & 1 & 4 \\ 3 & -1 & 1 & 6 \\ 4 & -1 & 2 & 7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & -9 \\ 0 & 1 & -1 & 22 \\ 0 & 2 & -2 & 33 \\ 0 & 3 & -2 & 43 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 4R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & -9 \\ 0 & 1 & -1 & 22 \\ 0 & 0 & 0 & -11 \\ 0 & 0 & 1 & -23 \end{bmatrix} \begin{matrix} R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 3R_2 \end{matrix}$$

$R_3 \leftrightarrow R_4$

$\rho(A) \neq \rho(A|B)$

Inconsistent and no solution.

b) $2\cos\alpha = x + \frac{1}{x}$

$2\cos\alpha x = x^2 + 1$

$\Rightarrow x = \cos\alpha \pm i\sin\alpha$

$2\cos\beta = y + \frac{1}{y} \Rightarrow y = \cos\beta \pm i\sin\beta$

$x^m = \cos m\alpha \pm i\sin m\alpha$

$\frac{1}{x^m} = \cos m\alpha - i\sin m\alpha$

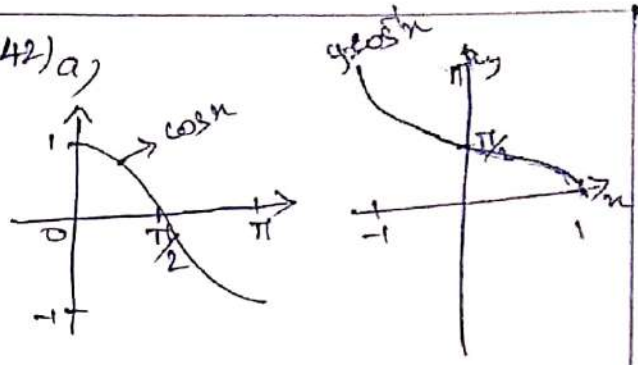
$\frac{x^m}{y^n} \pm \frac{y^n}{x^m} = 2\cos m\alpha$

$x^m y^n = \cos(m\alpha + n\beta) + i\sin(m\alpha + n\beta)$

$\frac{1}{x^m y^n} = \cos(m\alpha + n\beta) - i\sin(m\alpha + n\beta)$

$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

42) a)



b) $x^2 + y^2 + 2gx + 2fy + c = 0$

It passes through (1,1) (2,-1) & (3,2)

$2g + 2f + c = -2$

$4g - 2f + c = -5$

$6g + 4f + c = -13$

$f = -\frac{1}{2}, g = -\frac{5}{2}, c = 4$

$\therefore x^2 + y^2 - 5x - y + 4 = 0.$

43) a) Diagram.

$x^2 = -4ay$

$P(3, -2.5)$ passes through point

$a = \frac{9}{10}$

$Q(x_1, -7.5)$ also lies on the parabola.

$x_1^2 = 27$

$x_1 = 3\sqrt{3}$ //

H.O.D. Maths Depart.
S.R.V. Boys Hr.Sec.School,
Muthukalipatty - 637 401.
Rasipuram(TK), Namakkal(DT).

43)
b) Diagram.

$\hat{a} = \cos\alpha\hat{i} - \sin\alpha\hat{j}$

$\hat{b} = \cos\beta\hat{i} + \sin\beta\hat{j}$

$\hat{a}.\hat{b} = |\hat{a}||\hat{b}|\cos(\alpha+\beta) = \cos(\alpha+\beta)$ → ①

$(\hat{a}.\hat{b}) = (\cos\alpha\hat{i} - \sin\alpha\hat{j}).(\cos\beta\hat{i} + \sin\beta\hat{j})$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$ → ②

① & ②

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

44) a)
Vector Eqn:

$\vec{r} = \vec{a} + s\vec{u} + t\vec{v}$

$\vec{r} = (\hat{j} - 5\hat{k}) + s(2\hat{i} + 3\hat{j} + 6\hat{k}) + t(\hat{i} + \hat{j} - \hat{k})$

Cartesian Eqn:

$$\begin{vmatrix} x-0 & y-1 & z+5 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow 9x - 8y + z + 13 = 0$ //

b) $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx$ → ①

$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$I = \int_{-\pi}^{\pi} a^x \left(\frac{\cos^2 x}{a^x+1}\right) dx$ → ②

$2I = \int_{-\pi}^{\pi} \cos^2 x\,dx$ (even fn)

$\boxed{I = \pi/2}$

45) a) Diagram.

$\frac{dy}{dt} = -60$, $\frac{dz}{dt} = 20$

$z^2 = x^2 + y^2$ → ①

① ⇒ $2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\left(\frac{dy}{dt}\right)$

$x = 0.8$, $y = 0.6 \Rightarrow z = 1$

$\frac{dx}{dt} = 70$ Km/hr

b) $y = \begin{cases} \cos x & ; 0 \le x \le \frac{\pi}{2} \\ -\cos x & ; \frac{\pi}{2} \le x < \pi \end{cases}$

Diagram:

$A = \int_0^{\pi/2} \cos x\,dx + \int_{\pi/2}^{\pi} -\cos x\,dx$.

$A = 2$ sq. units.

H.O.D. Maths Department
S.R.V. Boys Hr.Sec.School,
Muthukalipatty - 637 401.
Rasipuram(Tk), Namakkal(Dt).

46)
a) Area = 196

side = 14

Volume $= x(14-x)^2$

$\therefore V = 196x + x^3 - 28x^2$

$V' = 196 + 3x^2 - 56x$

for max/min, $V' = 0$.

$\therefore x = \frac{49}{3}$, $x = \frac{7}{3}$

$V'' = 6x - 56$

when $x = \frac{49}{3}$, $V'' > 0$.

$x = 7/3$, $V'' < 0$.

∴ Volume is maximum when side is removed by $7/3$.

46) b)

$$M\frac{dv}{dt} = F - Kv$$

$$\frac{dv}{dt} + \frac{K}{M}v = \frac{F}{M}$$

$$I.F = e^{\int \frac{K}{M}dt} = e^{Kt/M}$$

Soln.

$$v\,e^{Kt/M} = \int \frac{F}{M} e^{Kt/M}\,dt + C$$

$$v\,e^{Kt/M} = \frac{F}{M}\, e^{Kt/M} \cdot \frac{M}{K} + C$$

$$v = \frac{F}{K} + C e^{-Kt/M}$$

$$t=0, v=0 \Rightarrow C = -\frac{F}{K}$$

$$v = \frac{F}{K}\left(1 - e^{-Kt/M}\right) //$$

47) a)

$$\frac{dT}{dt} = k(T-50)$$

$$\frac{dT}{T-50} = dt$$

$$\Rightarrow 50 - T = Ce^{kt}$$

(i) $t=0$, $T=70 \Rightarrow C=-20$

(ii) $t=2$, $T=60 \Rightarrow -10 = -20e^{2k}$

$$k = \frac{1}{2}\log\left(\frac{1}{2}\right)$$

$$50 - T = -20e^{\frac{t}{2}\log(\frac{1}{2})}$$

$$T = 50 + 20\left(\frac{1}{2}\right)^{t/2}$$

$T(t) = 98.6$ gives $t = -2.56$ i.e., 5.26 pm (app)

47) b)

x	0	1	2	3
$f(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$E(X) = \sum x_i f(x_i) = 3/2$$

$$E(X^2) = \sum x_i^2 f(x_i) = 3$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= 3 - \frac{9}{4} = \frac{3}{4}$$

Binomial;

$n=3$, $p=1/2$, $q=1/2$

$$\text{mean} = np = 3(1/2) = 3/2$$

$$\text{Var} = npq = 3(1/2)(1/2) = 3/4$$

B. Ramesh

H.O.D. Maths Department
S.R.V. Boys Hr.Sec.School,
Muthukalipatty - 637 401.
Rasipuram(Tk), Namakkal(Dt).